Appendix

# Appendix A

## Splitting the Helmholtz Equation

We may consider for instance the one dimensional case. Remind that the Helmholtz equation is:

|  |  |  |
| --- | --- | --- |
|  |  | (1) |

And the ersatz solution is:

|  |  |  |
| --- | --- | --- |
|  |  | (2) |

We plug (2) into (1) and search an expression for the second derivative term. If we derive a first time the solution (2) we obtain:

We then derive a second time:

Now we express the reflective index **Error! Reference source not found.** in terms of its real and imaginary part:

So equation (1) become now:

After having eliminating and separate real and imaginary part it becomes:

|  |  |  |
| --- | --- | --- |
|  |  | (3) |

|  |  |  |
| --- | --- | --- |
|  |  | (4) |

And this explains both **Error! Reference source not found.** and **Error! Reference source not found.**. To derive the higher dimension form from (3) and (4) see **Error! Reference source not found.**.

# Appendix B

## From one to two dimensions

We wish to give a proof sketch of how we allowed the two equations:

|  |  |  |
| --- | --- | --- |
|  |  | (5) |

|  |  |  |
| --- | --- | --- |
|  |  | (6) |

In the form suitable to higher dimensions:

|  |  |  |
| --- | --- | --- |
|  |  | (7) |

|  |  |  |
| --- | --- | --- |
|  |  | (8) |

In the following explanations, we will use the notion of directional derivative. This is generally done along the unit vectors of the standard basis: and. The proof is given for the two dimensional space but the argument may easily be extended to any higher dimension.

## Second order derivative and Laplacian

Let be a function of the two space variable vector:.

The terms that contain the dot product nullify. And it remains:

## Product of first order derivative

Let and be two function of the two space variable vector:.

The terms that contain the dot product nullify. And it remains: